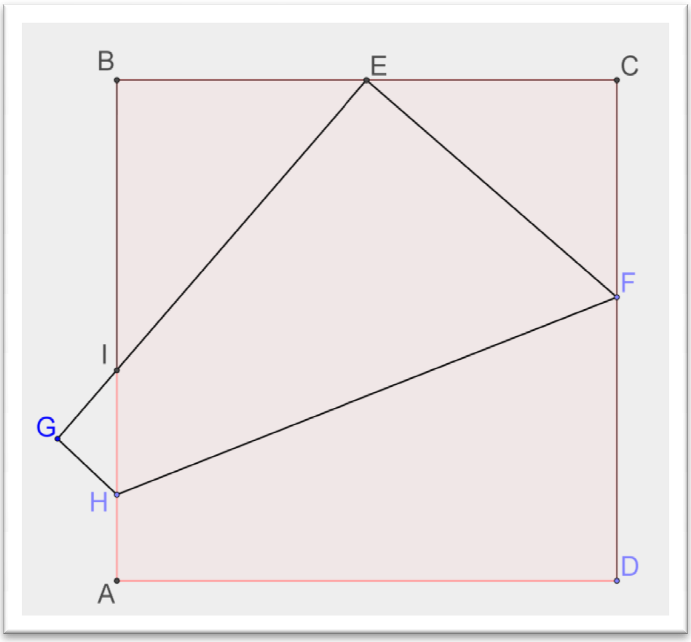
Simple Origami Problem



You are given a square paper of sides 24.
E is the mid-point of BC.

The piece of paper is folded so that the point D coincides with point E.

HF is the line of folding and point A goes to point G.

By considering the ΔCEF, find all line segments in the diagram.

Solution

Let $CF = x$

$$CE = \frac{1}{2} \times 24 = 12$$

$$EF = FD = 24 - x$$

By Pythagoras Theorem,

$$(24 - x)^2 = x^2 + 12^2$$

$$x^2 - 48\,x + 576 = x^2 + 144$$

$$\therefore x = 9$$

$$CF = x = 9$$

$$EF = FD = 24 - 9 = 15$$

Note that $\angle IEF = 90°$

$\angle BEI = 90° - \angle CEF = \angle CFE$

Hence $\Delta BEI \sim \Delta CFE$ (AAA)

$$\frac{BE}{CF} = \frac{EI}{FE} = \frac{IB}{EC} \Longrightarrow \frac{12}{9} = \frac{EI}{15} = \frac{IB}{12} \Longrightarrow \begin{cases} EI = 20 \\ IB = 16 \end{cases}$$

Hence, $\mathrm{IA} = 24 - \mathrm{IB} = 24 - 16 = 8$

Let $\mathrm{GH} = \mathrm{HA} = y$, $\mathrm{HI} = 8 - y$

Also, $\Delta\mathrm{GHI} \sim \Delta\mathrm{BEI}$ (AAA)

$$\frac{\mathrm{GH}}{\mathrm{BE}} = \frac{\mathrm{HI}}{\mathrm{EI}} = \frac{\mathrm{IG}}{\mathrm{IB}} \Longrightarrow \frac{y}{12} = \frac{8-y}{20} = \frac{\mathrm{IG}}{16}$$

$$\frac{y}{12} = \frac{8-y}{20} \Longrightarrow y = 3$$

Hence, $\mathrm{GH} = \mathrm{HA} = 3$, $\mathrm{HI} = 8 - 3 = 5$

Also, $\frac{3}{12} = \frac{\mathrm{IG}}{16} \Longrightarrow \mathrm{IG} = 4$

By Pythagoras Theorem,

$$\mathrm{HF}^2 = \mathrm{AD}^2 + (\mathrm{FD} - \mathrm{HA})^2$$

$$\mathrm{HF}^2 = 24^2 + (15 - 3)^2$$

$$\mathrm{HF} = 12\sqrt{5}$$

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